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Longitudinal quark polarization in transversely polarized nucleons

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Abstract

Accounting for transverse momenta of the quarks, a longitudinal quark spin asymmetry exists in a transversely polarized nucleon target. The relevant leading quark distribution $g_{1T}(x, k_T^2)$ can be measured in the semi-inclusive deep-inelastic scattering. The average k_T^2 weighted distribution function $g_{1T}^{(1)}$ can be obtained directly from the inclusive measurement of g_2.

Intrinsic transverse momentum (k_T) plays an important role in the quark *distribution functions* (DF's) used to describe a polarized nucleon [1,2]. For the leading (twist-two)

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part of the deep inelastic scattering cross section one already needs six DF's to describe the quark state in a polarized nucleon. They depend on x and k_T^2, which parametrize the quark momentum in a nucleon with momentum P, $k = x\,P + k_T$. We will adopt the notation of Ref. [2] for these "new" six independent DF's: f_1^q, g_{1L}^q, g_{1T}^q, h_{1T}^q, $h_{1L}^{q\perp}$, and $h_{1T}^{q\perp}$ (q denotes the quark flavor). For a polarized nucleon the spin vector is written as $S_N = \lambda\,P/M + S_T$, satisfying $\lambda^2 - S_T^2 = 1$. The probability, $\mathcal{P}_N^q(x,k_T^2)$, the longitudinal spin distribution, $\lambda^q(x,k_T)$, and the transverse spin distributions, $s_T^q(x,k_T)$, of the quark in a polarized nucleon are given by

$$\mathcal{P}_N^q(x,k_T^2) = f_1^q(x,k_T^2), \tag{1}$$

$$\mathcal{P}_N^q(x,k_T^2)\,\lambda^q(x,k_T) = g_{1L}^q(x,k_T^2)\,\lambda - g_{1T}^q(x,k_T^2)\,\frac{k_T\cdot S_T}{M}\,, \tag{2}$$

$$\mathcal{P}_N^q(x,k_T^2)\,s_T^q(x,k_T) \, = h_{1T}^q(x,k_T^2)\,S_T + \left[h_{1L}^{q\perp}(x,k_T^2)\lambda - h_{1T}^{q\perp}(x,k_T^2)\frac{k_T\cdot S_T}{M}\right]\frac{k_T}{M}, \tag{3}$$

These DF's have a clear physical interpretation: for example, g_{1T}^q describes the quark longitudinal polarization in a transversely-polarized nucleon. Such a polarization can be nonvanishing only if the quark transverse momentum is nonzero. This DF cannot be measured in *deep-inelastic scattering* (DIS) at leading order in $1/Q$. It can be measured in polarized *semi-inclusive deep-inelastic scattering* (SIDIS) as first shown in [3], where it appears as an azimuthal asymmetry. Measurements of the other "new" DF's were proposed in the doubly-polarized Drell-Yan process [1,2], and in the polarized SIDIS [3,4] using the so called Collins effect [5]. The quark fragmentation is described by two *fragmentation functions* (FF's): spin–independent and transverse–spin–dependent ones.

The "ordinary", $f_1^q(x)$, $g_1^q(x)$ and $h_1^q(x)$, and the "new" leading-twist DF's are related by k_T–integration

$$f_1^q(x) = \int d^2k_T\, f_1^q(x,k_T^2), \tag{4}$$

$$g_1^q(x) = \int d^2k_T\, g_{1L}^q(x,k_T^2), \tag{5}$$

$$h_1^q(x) = \int d^2k_T \left[h_{1T}^q(x,k_T^2) - \frac{k_T^2}{2M^2}h_{1T}^{q\perp}(x,k_T^2)\right]. \tag{6}$$

The DF $g_{1T}^q(x, k_T^2)$ does not contribute to $g_1^q(x)$, but it does contribute to the DF $g_T^q(x) = g_1^q(x) + g_2^q(x)$, which contributes at $\mathcal{O}(1/Q)$ in the inclusive polarized leptoproduction cross section [6]. A detailed discussion of the DF g_2^q is given in the recent review by Anselmino, Efremov and Leader [7].

In this letter we will be mainly concerned with the longitudinal quark spin distribution $\lambda^q(x, k_T)$ and the two DF's $g_{1L}^q(x, k_T^2)$ and $g_{1T}^q(x, k_T^2)$ describing it. Following Ref. [3], we first consider the polarized SIDIS in the simple quark-parton model. We will use the standard notation for DIS variables: l and l' are the momenta of the initial and the final state lepton; $q = l - l'$ is the exchanged virtual photon momentum; P (M) is the target nucleon momentum (mass), S its spin; P_h is the final hadron momentum; $Q^2 = -q^2$; $s = Q^2/xy$; $x = Q^2/2P \cdot q$; $y = P \cdot q/P \cdot l$; $z = P \cdot P_h/P \cdot q$. The reference frame is defined with the z-axis along the virtual photon momentum direction (antiparallel) and x-axis in the lepton scattering plane, with positive direction chosen along lepton transverse momentum. Azimuthal angles of the produced hadron, ϕ_h, and of the nucleon spin, ϕ_S, are counted around z-axis (for more details see Refs [3] or [8]). In this letter as independent azimuthal angles we will choose $\phi_h^S \equiv \phi_h - \phi_S$ and $\phi_l^S \equiv \phi_l - \phi_S$ and we will give cross-sections integrated over ϕ_l^S at fixed value of ϕ_h^S.

In leading order in $1/Q$ the SIDIS cross section for polarized leptons and hadrons has the form

$$\frac{d\sigma(\ell N \to \ell' h X)}{dxdydz\, d^2 P_{h\perp}} = \frac{2\pi\alpha^2}{Q^2\, y}\,[1 + (1-y)^2]\ \left(\mathcal{H}^0_{f_1} + D(y)\left[\lambda\,\mathcal{H}^0_{g_{1L}} + |S_T|\,\cos\phi_h^S\,\mathcal{H}^0_{g_{1T}}\right]\right), \tag{7}$$

where

$$D(y) = \frac{y(2-y)}{1 + (1-y)^2} \tag{8}$$

is the depolarization of the virtual photon with respect to the parent lepton. We do not consider here the cross section for unpolarized leptons and polarized hadrons which involves the structure functions $\mathcal{H}^S_{h_{1T}}$, $\mathcal{H}^S_{h_{1L}^\perp}$, and $\mathcal{H}^S_{h_{1T}^\perp}$ [3,4]. This single-polarized part of the SIDIS cross-section drops out after integration over ϕ_l^S in leading order in $1/Q$.

The structure functions $\mathcal{H}_f^0$ entering in Eqs (7) are given by quark-charge-square weighted sums of definite k_T-convolutions of the DF's and the well-known spin-independent FF $D_q^h(z,(P_{h\perp}-zk_T)^2)$. Taking into account the transverse momentum the latter depends on z and the transverse momentum squared of the produced hadron relative to the parent quark. Neglecting radiative corrections, the functions are independent of Q^2, however. The explicit form of the structure functions can be found in refs [3] or [8]:

$$\mathcal{H}_{f_1}^0 = \sum_q e_q^2 \int d^2k_T\, f_1^q(x,k_T^2) D_q^h(z,(P_{h\perp}-zk_T)^2), \tag{9}$$

$$\mathcal{H}_{g_{1T}}^0 = \sum_q e_q^2 \int d^2k_T \frac{\boldsymbol{k}_T\cdot\boldsymbol{P}_{h\perp}}{M\,|\boldsymbol{P}_{h\perp}|}\, g_{1T}^q(x,k_T^2) D_q^h(z,(P_{h\perp}-zk_T)^2), \tag{10}$$

$$\mathcal{H}_{g_{1L}}^0 = \sum_q e_q^2 \int d^2k_T\, g_1^q(x,k_T^2) D_q^h(z,(P_{h\perp}-zk_T)^2). \tag{11}$$

Note, that these structure functions include only the rather well studied unpolarized FF's, $D_q^h(z)$.

The target-longitudinal-polarization asymmetry is defined as

$$\mathcal{A}_L(x,y,z,P_{h\perp}) = \frac{d\sigma^{\rightarrow}-d\sigma^{\leftarrow}}{d\sigma^{\rightarrow}+d\sigma^{\leftarrow}}, \tag{12}$$

where $\rightarrow$ ($\leftarrow$) means longitudinal polarization, $\lambda = 1$ (-1) and $S_T = 0$. Analogously, the target-transverse-spin asymmetry is defined as

$$\mathcal{A}_T(x,y,z,P_{h\perp},\phi_h^S) = \frac{d\sigma^{\uparrow}-d\sigma^{\downarrow}}{d\sigma^{\uparrow}+d\sigma^{\downarrow}}. \tag{13}$$

with $\uparrow$ ($\downarrow$) denoting the transverse polarization of the target nucleon with $\lambda = 0$ and $|\boldsymbol{S}_T|$ $= 1$.

The phase space element in the transverse direction is $d^2P_{h\perp} = |\boldsymbol{P}_{h\perp}|\, d\,|\boldsymbol{P}_{h\perp}|\, d\phi_h^S$. Integrating $\mathcal{A}_L$ over ϕ_h^S we are left with the contribution proportional to $\mathcal{H}_{g_{1L}}^0$,

$$\int \frac{d\phi_h^S}{2\pi}\,\mathcal{A}_L = D(y)\,\frac{\mathcal{H}_{g_{1L}}^0}{\mathcal{H}_{f_1}^0}. \tag{14}$$

One can also define the asymmetry

$$\langle A_L\rangle\,(x,y,z) \equiv \frac{\int d^2P_{h\perp}(d\sigma^{\rightarrow}-d\sigma^{\leftarrow})}{\int d^2P_{h\perp}(d\sigma^{\rightarrow}+d\sigma^{\leftarrow})} = D(y)\,\frac{\sum_q e_q^2\, g_1^q(x)\, D_q^h(z)}{\sum_q e_q^2\, f_1^q(x)\, D_q^h(z)}. \tag{15}$$

This asymmetry was measured by the SMC collaboration [9] and provides the flavour analysis of the quark longitudinal-spin DF's in longitudinally polarized nucleon [10,11]. The future measurements are planned by the HERMES [12] and the HMC [13] collaborations.

The target-transverse-spin asymmetry is given by

$$\mathcal{A}_T(x,y,z,P_{h\perp},\phi_h^S) = D(y)\,\cos\phi_h^S\,\frac{\mathcal{H}^0_{g_{1T}}}{\mathcal{H}^0_{f_1}} \tag{16}$$

and can in principle be disentangled measuring the asymmetry at different values of ϕ_h^S and performing a Fourier analysis. For example, let us integrate Eq. 16 weighted by $\cos\phi_h^S$ over ϕ_h^S. We obtain

$$\int_0^{2\pi}\frac{d\phi_h^S}{2\pi}\,\cos\phi_h^S\,\mathcal{A}_T(x,y,z,P_{h\perp},\phi_h^S) = \frac{1}{2}D(y)\frac{\mathcal{H}^0_{g_{1T}}}{\mathcal{H}^0_{f_1}}. \tag{17}$$

It is useful to define the transverse-spin asymmetry weighted with $\boldsymbol{S}_T\cdot\boldsymbol{P}_{h\perp}/M = (|\boldsymbol{P}_{h\perp}|/M)\cos\phi_h^S$,

$$\begin{aligned}\langle\frac{|\boldsymbol{P}_{h\perp}|}{M}\cos\phi_h^S\,A_T\rangle\,(x,y,z) &= \frac{\int d^2P_{h\perp}\frac{|\boldsymbol{P}_{h\perp}|}{M}\,\cos\phi_h^S\,(d\sigma^\uparrow - d\sigma^\downarrow)}{\int d^2P_{h\perp}(d\sigma^\uparrow + d\sigma^\downarrow)}\\ &= zD(y)\,\frac{\sum_q e_q^2\,g_{1T}^{q\,(1)}(x)\,D_q^h(z)}{\sum_q e_q^2\,f_1^q(x)\,D_q^h(z)},\end{aligned} \tag{18}$$

where

$$g_{1T}^{q\,(1)}(x) = \int d^2k_T\,\frac{\boldsymbol{k}_T^2}{2M^2}\,g_{1T}^q(x,k_T^2). \tag{19}$$

Note, that relations 15 and 18 are valid for any k_T–dependence of DF's and FF's. In principle, it is possible to separate contributions from different quark flavours by measuring the asymmetry 18 for different produced hadrons in the same way as proposed in [10,11].

Next we turn to a quantitative estimates of the asymmetries, starting with the longitudinal asymmetry. For this we consider the production of π^+-mesons on the proton. The dominant contribution will come from scattering on the u-quark. In order to estimate $\langle A_L\rangle$ we use the parametrization of Brodsky, Burkardt and Schmidt (BBS) [14] for g_1^u and f_1^u. The result,

$$\frac{1}{D(y)}\langle A_L\rangle(x,y,z) \approx \frac{g_1^u(x)}{f_1^u(x)}, \tag{20}$$

is shown in Fig. 1.

For an estimate of the transverse asymmetry we need the DF's $g_{1T}^q(x,k_T^2)$. In contrast to the k_T-integrated DF's $f_1^q(x)$ and $g_1^q(x)$ there is no measurements of the function $g_{1T}^q(x,k_T^2)$. As is shown in Ref. [6,8] the $(\boldsymbol{k}_T^2/2M^2)$–weighted k_T-integrated function $g_{1T}^{q\,(1)}(x)$, which appears in Eq. 18 is directly related to the DF $g_2^q(x)$,

$$g_2^q(x) = \frac{d}{dx}\, g_{1T}^{q\,(1)}. \tag{21}$$

This relation just follows from constraints imposed by Lorentz invariance on the antiquark-target forward scattering amplitude and the use of QCD equations of motion for quark fields. We will use this relation for our quantitative estimates. We note that the effects of higher order QCD corrections for the transverse momentum dependent functions, however, require further investigation [15]. For the function g_2 the QCD corrections have been extensively studied [16].

In our first estimate for the transverse asymmetry (Eq. 18) we use recent data on g_2 and the relation in Eq. 21. Such data are available from the SMC collaboration [19] and the E143 collaboration at SLAC [20]. Particularly the latter data allow a rough estimate of the function,

$$g_{1T}^{(1)}(x) = \frac{1}{2}\sum_q e_q^2 g_{1T}^{q\,(1)}(x) = \int d^2k_T \frac{\boldsymbol{k}_T^2}{2M^2}\, g_{1T}(x,k_T^2) = -\int_x^1 dy\, g_2(y). \tag{22}$$

The result obtained by averaging the two sets of data at different angles and adding statistical and systematic errors quadratically, is shown in Fig. 2.

Our second estimate for this distribution function comes from the representation for $g_2^q(x)$ in terms of other k_T-integrated functions. For $g_T^q = g_1^q + g_2^q$ one has

$$g_T^q(x) = \int_x^1 dy \frac{g_1^q(y)}{y} + \frac{m_q}{M}\left[\frac{h_1^q(x)}{x} - \int_x^1 dy \frac{h_1^q(y)}{y^2}\right] + \tilde{g}_T^q(x) - \int_x^1 dy \frac{\tilde{g}_T^q(y)}{y}, \tag{23}$$

where m_q is the quark mass, and $\tilde{g}_T^q$ is the so called interaction-dependent part of the DF $g_T^q(x)$. The term $(m_q/Mx)h_1^q(x)$ in the *rhs* of Eq. 23 was found many years ago by Feynman

[17] and represents the contribution of the transverse spin distribution to $g_T(x)$. The most well-known contribution in Eq. 23 is the first term found by Wandzura and Wilczek [18]. Using Eq. 21 an estimate of $g_{1T}^{(1)}(x)$ is obtained from Eq. 23, keeping only the first term (Wandzura-Wilczek) as this does not contradict the data. In that case $g_T(x) = g_T^{WW}(x)$ where

$$g_T^{WW}(x) = g_1(x) + g_2^{WW}(x) = \int_x^1 dy\, \frac{g_1(y)}{y}, \tag{24}$$

leading to

$$g_{1T}^{(1)WW}(x) = -\int_x^1 dy\, g_2^{WW}(y) = x \int_x^1 dy\, \frac{g_1(y)}{y} = x\, g_T^{WW}(x). \tag{25}$$

We use the parametrization of DF's from Ref. [14]. The result is shown as the curve in Fig. 2. Using other parametrizations for g_1 does not substantially change this result.

Assuming the u-quark dominance for the π^+ production on the proton, the estimate for the transverse spin asymmetry,

$$\frac{1}{z\, D(y)} \langle \frac{|\boldsymbol{P}_{h\perp}|}{M} \cos \phi_h^S\, A_T \rangle\, (x, y, z) \approx \frac{g_{1T}^{u\,(1)}(x)}{f_1^u(x)}, \tag{26}$$

can be obtained (see Fig. 3).

In this letter we have considered the azimuthal asymmetry in 1-particle inclusive polarized leptoproduction. The longitudinal spin asymmetry averaged over the transverse momenta of the produced hadrons gives independent ways to study the polarized quark distributions as has been pointed out before [10,11]. As we have shown the transverse spin asymmetry provides information on the quark-longitudinal spin distribution in a transversely polarized target, the DF $g_{1T}^q(x, k_T^2)$. This information appears in a $\cos(\phi_h - \phi_S)$ asymmetry for the produced particles. A Fourier analysis of this asymmetry weighted with the modulus of the transverse momentum of produced particles, gives the k_T–integrated and $\boldsymbol{k}_T^2/2M^2$–weighted function $g_{1T}^{q\,(1)}(x)$ which is at tree-level directly related to $g_2^q(x)$. This provides an alternate way of obtaining the latter DF, although a careful analysis of the QCD corrections is needed.

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FIGURES

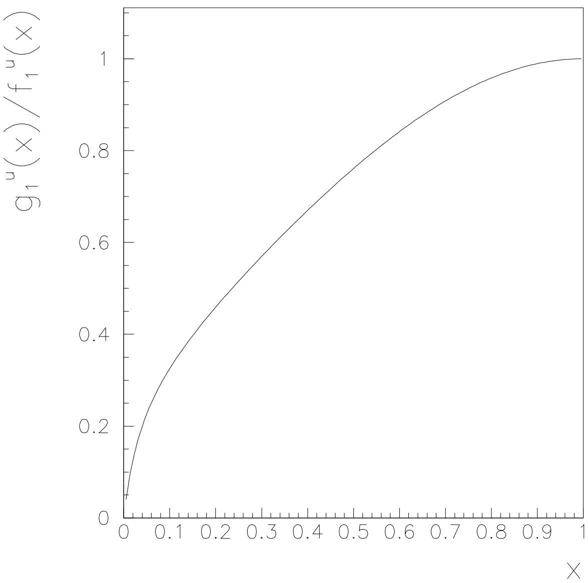

FIG. 1. The longitudinal spin asymmetry (Eq. 20) as function of x with BBS-parametrization.

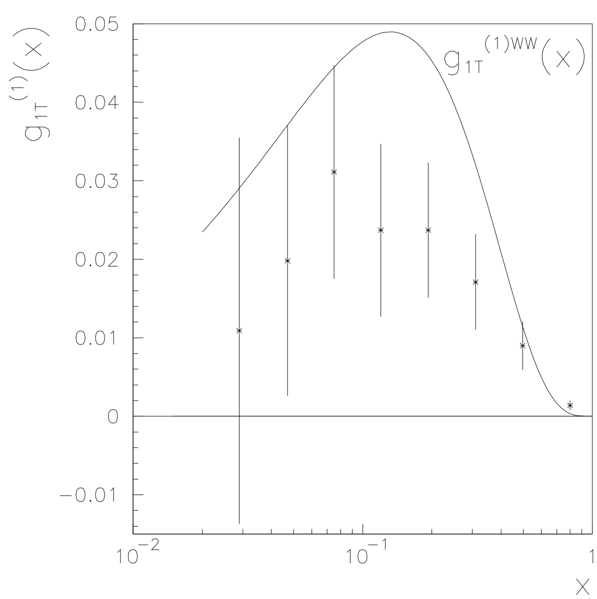

FIG. 2. The function $g_{1T}^{(1)}(x)$ as obtained from E143 data using Eq. 22 or from the BBS-parametrizations for g_1 using Eq. 25.

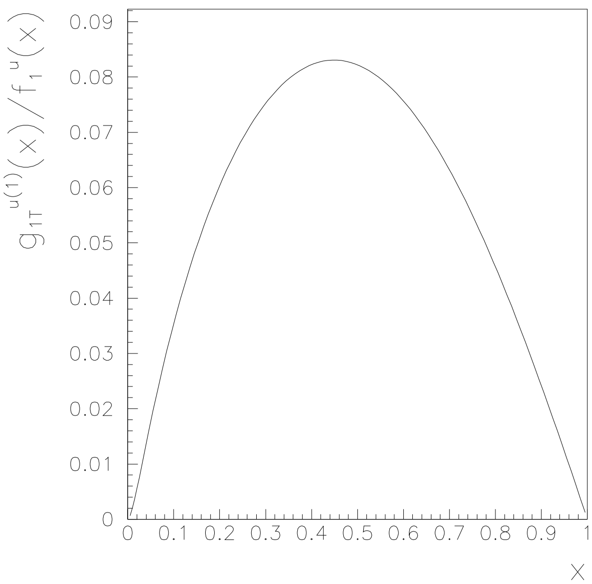

FIG. 3. The transverse spin asymmetry (Eq. 26) as function of x estimated from the BBS-parametrization for g_1 using Eq. 25.